SPO’s Standard CGE Model: TURKCGE
ALGEBRAIC EQUATIONS

I. Within-period system of equations

1) Price System

Composite price

\[ PC_i = (1 - tva_i) \left[ P_i^D \left( \frac{DC_i}{CC_i} \right) + P_i^M \left( \frac{M_i}{CC_i} \right) \right] \]

Import price

\[ P_i^M = (1 + t_m_i)P_i^W \]

Output price

\[ PX_i = \left[ P_i^D \left( \frac{DC_i}{XS_i} \right) + P_i^E \left( \frac{E_i}{XS_i} \right) \right] \]

Net (value added) price

\[ PVA_i = PX_i - t_m_i - \sum_j a_{ji}XS_jPC_j \]

2) Production Technology

Gross output supply

\[ X_i^S = \overline{A}K_i^\alpha L_i^\beta I_i^{(1-\alpha-\beta)} \]

Intermediate Input Demand in Sector i

\[ INT_i = \sum_j a_{ji}X_j^i \]
3) Labor Markets

To obtain labor demand, marginal productivity is equated with the real wage rate:
\[
\left( \frac{\partial X}{\partial L} = \frac{w}{PVA} \right)
\]

for skilled labor
\[
\beta \cdot VA \cdot X^s = (1 + tw)\bar{w}_s \bar{L}_s^D \quad \text{for} \ i \not\in \text{public services}
\]

for unskilled labor
\[
(1 - \alpha - \beta)PVA = w_{us} L_{us}^D
\]

Labor market clearing: Fixed nominal wages for skilled and flexible for unskilled labors
\[
w_s = \bar{w}_s \quad \Rightarrow \quad \text{UNEMP}_s = L_s^r - \sum_i L_i^D
\]
\[
\sum_i L_{us,i}^D = L_{us}^r + \text{UNEMP}_s
\]

4) Income Generation

Enterprise profits
\[
RP_i = PVA_i X_i^s - (1 + tw)\bar{w}_s L_{si}^D - w_{us} L_{usi}^D
\]

After-tax (net) profit income of enterprises
\[
YEnet = (1 - t_k) \sum_i RP_i
\]

Transfer of enterprise income to households (dividends)
\[
EtrHH = YEnet - \sum_i NF_{iG} - \text{trrow} \sum_i (1-tk)RP_i + GtrSEE + r^D \text{DomDebt}^G - r^E \text{ForDebt}^E + \text{ForBOR}^E
\]
Transfer of enterprise income to public sector (public sector net factor income)

\[ NFI_i^G = \text{shr}_g R_P \]

Profit transfers to the rest of the world

\[ \text{trrow} \sum (1 - tk) R_P \]

Private household net labor income

\[ YHW_{\text{net}} = (1 - \text{sstax}_i) \sum L_{S,j} + w_{US} \sum L_{US,j} \]

Total Private Income

\[ \text{totYHH} = YHW_{\text{net}} + \text{EtrHH} + \text{GtrHH} + \text{SSltrHH} + \text{ROWtrHH} \]

Net Private Income (Private Disposable Income)

\[ YH_{\text{net}} = (1 - ty) \text{totYHH} \]

5) Public Sector Balances

Public sector aggregate revenues

\[ \text{GREV} = \sum t_{ni} PX_i \cdot XS_i + \sum t_{n1} P_{\text{nom}} M_i + \sum t_{va_i} PQ_iCC_i + ty \cdot \text{totYHH} + t_K \sum R_P + \sum NFI_i^G \]

Public Savings

\[ \text{GSAV} = \text{GREV} - GCON - r^F \text{ForDebt}^G - r^D \text{DomDebt}^G - \text{GtrHH} - \text{GtrSEE} - \text{GtrSSI} + \text{ForBor}^G \]

Government fiscal policy (primary balance objective)

\[ \text{PRIMBAL} = \text{GREV} - GCON - GINV - \text{GtrHH} - \text{GtrSEE} - \text{GtrSSI} \]

\[ \text{PRIMBAL} = \text{prbrat} \cdot \text{GDP} \]

Fiscal closure (choose either of the following, leaving the other free)
\[ GCON = gcr \cdot GDP \]
\[ GINV = gir \cdot GDP \]

Social Security Institutions
Revenues

\[ revSSI = (tw + sstax_i) \sum_i L_{si,j} \]

Government transfers to SSIs

\[ GtrSSI = SSIttHH - revSSI \]

6) Financial Accounts

Private savings

\[ PSAV = s^P YHnet \]

Saving-Investment balance (Walras Law)

\[ PSAV + GSAX + CAdef = PINV + GINV \]

7) Sectoral Demands

Private consumption by sectors

\[ CD_i = clex_i \cdot \frac{PRIVCON}{PC_i} \]

where \( PRIVCON = (1 - s^P)YHnet \)

Private investment demand (by sector of origin)

\[ IDP_i = iplex_i \cdot \frac{PINV}{PC_i} \]
Government consumption by sectors

\[ GD_i = gles_i \cdot \frac{GOVCON}{PC_i} \quad i \notin \text{public services} \]

\[ GD_{PUBSERV} = w_s L^D_{S,PUBSERV} \]

Government investment demand (by sector of origin)

\[ IDG_i = igles_i \cdot \frac{GINV}{PC_i} \]

8) Rest of the World

\[ CAdf = \sum P_i^w E_i + ROWtHH + ForBor^E + ForBor^G \\
- \left[ \sum P_i^w M_i + trow \sum (1 - t)RP_i + r^E ForDebt^E + r^F ForDebt^G \right] \]

Determination of export supply

\[ X^*_i = CET(DC_i, E_i) \]

with export-domestic good ratio:

\[ \frac{E_i}{DC_i} = \left( \frac{P_i^w}{PD_i} \right)^\gamma \frac{\chi}{(1 - \chi)} \]

Determination of import demand

\[ CC_i = CES(DC_i, M_i) \]

with import-domestic good ratio:

\[ \frac{M_i}{DC_i} = \left( \frac{1 + m_i}{PD_i} P_i^w \right)^\varepsilon \frac{\delta}{(1 - \delta)} \]
9) Market Clearing

\[ CC_i = CD_i + GCON_i + IDP_i + IDG_i + INT_i \]

II. Dynamics

1) Evolution of government and private debt

*Government budget deficit (Public Sector Borrowing Requirement-PSBR)*

\[ PSBR = GREV - GCON - GINV - r^F ForDebt^G - r^D DomDebt^G - GtrHH - GtrSEE - GtrSSI \]

*Government’s Foreign Borrowing is a ratio of PSBR:*

\[ ForBor^G = (gfborrat)PSBR; \text{ thus } DomBor = (1 - gfborrat)PSBR \]

*Government Domestic Debt:*

\[ DomDebt_{t+1} = DomDebt_t + DomBor_t \]

*Government Foreign Debt*

\[ ForDebt^G_{t+1} = ForDebt^G_t + ForBor^G_t \]

*Private foreign debt*

\[ ForDebt^P_{t+1} = ForDebt^P_t + ForBor^E_t \]

2) Rules to update exogenous variables and policy variables:

Increase sectoral capital stocks by investments (by sector of destination) net of depreciation
Increase labor supplies by population growth
Increase technology indexes by TFP rates

The following variables are updates as a fixed ratio to the expenditures of respective units:
*ForBor^E* and *ForBor^G*, GtrHH, GtrSEE

The following are updated as a ratio to the GDP:
GtrSSI, ROWtrHH, SSLtrHH