Lecture 1

THE 2X2 SMALL OPEN ECONOMY MODEL IN GAMS

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1. An Open 2x2 Ricardo-Viner General Equilibrium Model with Sector Specific Capital.

2.The GAMS Code of the Model.

3.The Walras' Law, Normalization and the Exchange Rate.

4.The Production Sector and the Neo-classical Properties of the General Equilibrium System with Sluggish Capital Adjustment over Time.

1.1 A 2x2 Small Open Economy Model

In this section I first present an analytical model of an abstract model of an abstract two-sector, two-factor (2x2) economy. The economy is assumed to be *small* in the world markets with both goods being traded. There are two consumption goods, agricultural (A), and urban (N). The consumption goods are assumed to be produced along a CES technology obeying *neo-classical* hypotheses. Each sector uses capital, K, and labor, L, as primary inputs. It is assumed that the first sector is more labor intensive.

Labor is treated perfectly mobile across sectors. Capital, however, is sector-specific, and once installed, each sector is assumed to utilize its capital endowment up to the amount available. This specification gives rise to differences of the "profit rates" across sectors. In the later stage of the model, we hypothesize an environment where capital will move *sluggishly* out of the low-profit sector to the high profit-sector, until when profit rates are equalized.

There are two households, corresponding to factor ownership: workers and capitalists. As an initial condition, we assume that the worker household is endowed with 10 units of labor; and the capitalist household is endowed with 15 units of physical capital. The wage rate of 0.289 clears the labor market, and the capital market is in equilibrium with profit rates being at equal values initially at

0.336. Workers supply all their labor endowments to production activities, hence they do not have leisure considerations. Both households have Cobb-Douglas type preferences over the two commodities.

Initially the A-sector is assumed to be in excess demand (imported), and the N-sector has excess supply (exported). World terms of trade is set at unity. The complete list of equations are presented in Table 1.

Table 2.1: Algebraic Equations of the 2x2 Small Open Economy Model

Prices: $P_i = ER.PW_i$

FACTOR MARKETS

Production: $Q_i^s = A_i[\alpha_i K_i^{-P} + (1-\alpha_i)L_i^{-P}]^{-1/P_i}$

Labor demand: $L_i^D = [(P_i / W)(1-\alpha_i)A_i^{-P}]^{\sigma} Q_i^S$,

$$\text{With} \quad \sigma_i = \frac{1}{(1+P_i)}$$

Labor market equilibrium: $\sum L_i^D = \overline{L}^S$

Capital is sector specific: $K_i = \overline{K}_i$

Wage income: $Y^W = \sum W.L_i^D$

Capitalist income: $Y_K = \sum (P_i Q_i^S - W.L_i^D)$

COMMODITY MARKET S

Consumer demand: $Q_{h,i}^D = \beta_{h,i}\left[\frac{Y_h}{P_i}\right]$

Import (excess demand): $Q^M = \sum Q_{h,A} - Q_A^S$

Export (excess supply): $Q^E = Q_N^S - \sum Q_{h,N}^D$

Trade balance (Walras' Law): $P_A^W Q^M - P_N^W Q^E = 0$

1.2. The 2x2 Model in the GAMS Syntax

In this section we introduce the GAMS code for solving our simple general equilibrium model in a PC. The acronym GAMS, stands for the *General Algebraic Modeling System;* and is described in detail in Brooke, Kendrick and Meeraus (1988). In what follows, we will introduce the main principles and components of the program, referring the reader to the main documentation for further expertise.

Table 2.2 identifies the general structure of the GAMS program to implement the simple model. While the sequencing of the commands may vary from model to model, the structure displayed in Table 2.2 is quite straightforward, and we will be following this routine in our foregoing discussion.

First we introduce the SETS distinguished in our simple model. We identify three indexes: HH for households: workers and capitalists; I for commodities: A for agricultural, and N for the industrial; and TP for time periods, to be utilized in the "dynamic" stage of our modeling effort. We distinguish a subset, T of TP, as "the current" time period, which will assume a particular realization over the range of time periods, 1 through 50. The GAMS commands of the foregoing run as follows: :

```
*
***SET DEFINITIONS
 SET
 HH INDIVIDUALS /WHH WORKER HOUSEHOLD /
                         KHH CAPITALIST HOUSEHOLD /

 I GOODS /  A   AGRICULTURAL GOOD
N INDUSTRIAL GOOD /

 TP TIME PERIODS /0*50/
T(TP) CURRENT TIME PERIOD
```

The elements of each set are introduced by use of slashes ("I"), and the SET identification is closed with the use of the ubiquitous ";". The specific element that the index T will realize is to be introduced below.

Table 2: General Structure of the GAMS Program

Component	Description
SETS	Declares the indexes of the model
PARAMETERS	Declares the constants, parameters and the dummy variables to be used in the program. The elements under this component are described by a brief descriptive sentence following the declaration statement, Multi-dimensional parameter arrays are declared with the TABLE command, with the "." representing any item that can be freely red into the TABLE.
INITIAL DATA	Data is introduced using the TABLE and SCALAR commands.
CALIBRATION	Data is used to generate the structural parameters and the shift variables of the algebraic model equations. The purpose here is to impose the algebraic system of equations onto the data, so that the first "solution" of the model will reflect exactly the "base-year" SAM data.
VARIABLES	All variables, endogenous, or exogenous are declared here with brief descriptive identifiers.
VARIABLE INITIALIZATION	The Variables are initialized with the base-year data values. This enables the algorithm to start its search for equilibrium from a "sufficiently close" data point.
EQUATIONS	The algebraic equations of the model are declared with descriptive statements.
EQUATION SPECIFICATIONS	The declared EQUATIONS are specified in the GAMS language.
CLOSURE AND RESTRICTIONS	Exogenous variables are "fixed" using the .FX appends. Bounds are introduced using .LO and .UP. The overall macro closure recognized in the model is implemented here.
SOLVE AND DISPLAY	The MODEL statement declares the model by specifying which EQUATIONS form the MODEL. SOLVE statement commands GAMS to interface with the algorithm to search for a numerical solution. The solution values are inputted to the display tables and are documented using the DISPLAY command.

The parameters and the pseudo-variables to carry the initial values of the endogenous variables of the model are declared under PARAMETERS. Constants are declared using the SCALAR command, with data being entered between slashes such as /#.###/). Multi-dimensional parameters are declared by the TABLE command, with the elements of the TABLE being declared according to the indexes ordered in parentheses. If no SET index is described, the star ("*") can be used to declare "free" elements to be used in that TABLE. The GAMS code of this section is as follows:

```

***PARAMETER DEFINITIONS
PARAMETER
AD(I) SHIFT PARAMETER IN PROD FUNC
ALFA(I) CES FUNCTION SHARE PARAMETER
CLES(I,HH) SECTORAL CONSUMPTION DEMAND BY HOUSEHOLDS
PW(I) WORLD PRICES
RHO(I) PRODUCTION EXPONENTIAL PARAMETER
SIGP(I) PRODUCTION FUNCTION ELASTICITY OF SUBSTITUTION

*DUMMIES TO HOLD INITIAL DATA
IGO(HH) HOUSEHOLD WEIGHTS IN THE SOCIAL WELFARE FUNCTION
KO(I) CAPITAL DEMAND IN SECTOR I
KSTAR(HH) HOUSEHOLD CAPITAL ENDOWMENT
LDO(I) SECTORAL LABOR DEMANDS
LSTAR(HH) HOUSEHOLD LABOR ENDOWMENT
PO(I) DOMESTIC PRICES
QSO(I) COMMODITY SUPPLIES
RPO(I) SECTORAL PROFITS
RKO(I) SECTORAL PROFIT RATE
VO(HH) HOUSEHOLD INDIRECT UTILITIES
WLO(I) SECTORAL WAGES
```

```
YO(HH) HOUSEHOLD INCOME
;

SCALAR
WO NOMINAL WAGE RATE
QXO EXPORTS
QMO IMPORTS
ERO EXCHANGE RATE /1.0/


*TABLES TO DISPLAY RESULTS
PARAMETER
HOUSE(*,HH,TP) HOUSEHOLD RESULTS
PRODUCT(*,I,TP) PRODUCT MARKET RESULTS
SCALRES(*,TP) SCALAR RESULTS
RK(I) PROFIT RATE AT PERIOD T
QDT(I,HH,TP) HOUSEHOLD DEMAND TABLE
;
```

Material Balances for the Simple General Equilibrium Model

		Activities		Commodities		Factors		Households			
		1.Agr	2.Ind	3.Rural	4.Urban	5.Labor	6.Capital	7.Worker	8.Capitalist	9.ROW	SUM
Activities	1. Agriculture			2.206	0.000						2.206
	2. Industry			0.000	5.721						5.721
Commodities	3. Rural							2.167	2.519	0.000	4.686
	4. Urban							0.722	2.519	2.480	5.721
Factors	5. Labor	0.986	1.903								2.889
	6. Capital	1.220	3.818								5.038
Households	7. Workers					2.889	0.000				2.889
	8. Capitalist					0.000	5.038				5.038
ROW	9. Row			2.480	0.000						2.480
TOTALS		2.206	5.721	4.686	5.721	2.889	5.038	2.889	5.038	2.480	

The data is introduced, mostly using TABLEs. The data usually come from an underlying Social Accounting Matrix (SAM), and should, desirably, be in initial equilibrium. The initial data points of our simple model are represented in the GAMS syntax as:

```

***INITIAL DATA FROM SAM TABLE

TABLE MATBAL(*,I) MATERIAL BALANCES

        A       N
PW      1       1
WLO     0.986   1.903
QSO     2.206   5.721
LDO     3.413   6.587
KO      3.636   11.364
SIGP    0.8     0.4
;

TABLE HHINC(*,HH) HOUSEHOLD PARAMETERS

        WHH     KHH
YO      2.889   5.038
LSTAR   10.0
KSTAR           15.0
IGO     1. O    1. O
;

TABLE QDO(I,HH) HOUSEHOLD CONSUMPTION

        WHH     KHH
A       2.167   2.519
N       0.722   2.519
;


```

We *calibrate* the model by using the algebraic equation system and the initial data points of the model. Thus, using the base-year data as a point estimate, the outcome of this exercise reveals the values of all the structural shift and share parameters of the model equations. The idea is to "force" the program to choose those parameter values so that the initial solution of the program is an exact replica of the base-year benchmark equilibrium data set compiled in the SAM accounts. These ideas are formulated in the GAMS code as follows:

```
*##MODEL CALIBRATION
PW(I) = MATBAL{"PW" ,I);
PO(I) = ERO*PW{I);
LDO(I) = MATBAL{"LDO" ,I) ;
KO{I) = MATBAL("KO",I) ;
QSO(I) = MATBAL{"QSO" ,I) ;
WLO(I) = MATBAL(""WLO", I) ;
LS TAR (HH) = HHINC("LSTAR",HH) ;
KSTAR(HH) = HHINC("KSTAR",HH) ;
*FIRST CALCULATE THE WAGE RATE
WO = SUM{I, WLO(I) ) / SUM(HH, LSTAR(HH)) ;
*WE CALCULATE SECTORAL PROFITS BY DEDUCTING THE WAGE PAYMENTS
*FROM SECTORAL VALUE ADDED USING EVLER' S LAW (OBSERVE THAT THE
*PRODUCTION FUNCTION IS CRTS).
RPO ( I ) = PO ( I ) *QSO ( I) - WLO ( I) ;
*THEN WE CALCULATE THE PROFIT RATE
RKO(I) = RPO(I)/KO(I) ;
*##CALIBRATE THE PRODUCTION FUNCTION: CES(K,L)
SIGP(I) = MATBAL("SIGP",I);
RHO(I) = (l/SIGP(I))-l;
ALFA{I) = (KO(I)/LDO(I))**{-l-RHO(I))*{WO/RKO(I)) ;
ALFA{I) = 1/(1+ALFA(I)) ;
AD(I) = QSO(I) / ((ALFA(I)*KO(I)**(-RHO(I)) + (l-ALFA(I))*LDO(I)
        **(-RHO{I)))**(-l/RHO{I))) ;
DISPLAY AD, ALFA, RHO;
```

```
148 *##CALCULATE HOUSEHOLD CONSUMPTION PARAMETERS
149 YO(HH) = HHINC{"YO",HH) ;
150
151 CLES(I,HH) = PO{I)*QDO{I,HH) / YO(HH) ;
152
153 DISPLAY CLES ;
154
155 *##(INDIRECT) UTILITY LEVELS
156 IGO(HH) = HHINC{"IGO",HH);
157
158 VO(HH) = PROD{I, QDO{I,HH)**CLES(I,HH) ) ;
159
160
161 *##EXPORTS AND IMPORTS
162 QXO = QSO{"N") - SUM(HH, QDO("N",HH)) ;
163 QMO = SUM(HH, QDO{"A",HH)) - QSO("A") ;
```

We first read in available data from the TABLE's and load the dummy variables with their corresponding data points. (Lines 113 through 121). Observe that by setting the base-year prices at unity, we choose the initial price index at unity, and treat all base-year *nominal* quantities as real quantities, expressed in the base-year prices. This accounting procedure is especially helpful in later simulations *of* the model, to distinguish between real versus nominal changes across the variables.

Next we calculate the *wage* rate by dividing the total wage payments by aggregate employment to get wage earnings per unit *of* labor (line 125). Assuming constant returns to scale production technologies, factor payments exhaust he total sectoral value added. We get aggregate profits as a residual value added, by subtracting wage costs.

The CES production technology is calibrated to the production and employment data in lines through 137 to 144. In this process the Elasticity *of* substitution is given from "outside", desirably from econometric estimation. In lines 140 and 141, α_i of the CES equation is calibrated using the first order conditions for labor employment to maximize producer profits; while the shift parameter, A_i, is calibrated in lines 143 to 144. The values *of* these parameters are DISPLAYed in line 146.

Household consumption shares by sectors are found in line 151, under the assumption that the consumer preferences are Cobb-Douglas (line 158). Finally excess demand equations are implemented for calibrating the initial data points for exports (162) and imports (163).

Variable are declared under line 167 using the POSITIVE VARIABLE command, and under line185, using the simple VARIABLE command. The first usage ensures ha the corresponding values will always be bounded from below during the solution search procedure, as economists do not like to work with negative quantities.

```
164
165
166 ***VARIABLE DEFINITIONS
167 POSITIVE VARIABLE
168 PINDEX   PRICE INDEX
169 ER       NOMINAL EXCHANGE RATE (CONVERSION FACTOR)
170 P(I)     DOMESTIC PRICES
171 RP(I)    SECTORAL PROFITS
172 QS(I)    REAL OUTPOT SUPPLIES
173 LD(I)    SECTORAL LABOR DEMANDS
174 K(I)     SECTORAL CAPITAL DEMANDS
175 W NOMINAL WAGE RATE
176
177 Y(HH)    HOUSEHOLD INCOME
178 V(HH)    INDIRECT HOUSEHOLD UTILITY
179 IG(HH)   HOUSEHOLD WEIGHTS ON THE SOCIAL WELFARE FUNCTION
180
181 QD(I,HH) REAL HCUSEHOLD DEMAND
182
183 ;
184
185 VARIABLE
186 OMEGA    SOCIAL WELFARE FUNCTION VALUE
187 QX       REAL EXPORTS (EXCESS SUPPLY)
188 QM       REAL IMPORTS (EXCESS DEMAND)
189 ;
190
```

The variable in line 186 (OMEGA) is a cooked-up variable to initiate the non-linear solver algorithm. This variable is optimized under the SOLVE statement (line 327 below). However, since the Walrasian system is a square system of simultaneous equations, with the number of *independent* equations equal to that of the independent variables, the algorithm searches for an "optimum" value for OMEGA with no avail. It ought to declare "equilibrium solution" once all the equation restrictions are satisfied, and it cannot improve any further. In this respect, for models of this class, specification of OMEGA is entirely neutral on the solution values. However, for models _not in the Walrasian square system tradition, the way OMEGA is defined will be important, as the algorithm will try to optimize it treating the problem as one of "constrained optimization". Here we defined OMEGA as the weighted sum of the indirect utility levels of the private households. Note that the weights, IG_{hh} ,are entirely exogenous, and there is no mechanism in the model for private households to affect them. Such a mechanism would turn the model into a non-square system, bringing it closer to the models discussed in the endogenous political economy literature. [Cite our work, and our friends here].

We declare EQUATIONs by appending specific names to each class of algebraic equations. If preferred the equation names can be numbers, corresponding to the order of presentation in an accompanying text:

```
***EQUATION DEFINITIONS
EQUATION
INDEX PRICE NORMALIZATION RULE
PRICE DOMESTIC PRICES
PRODUCTION SUPPLY OF GOODS
LABOR DEMAND FOR LABOR
LMEQL LABOR MARKET EQUILIBRIUM
WINC WORKER HOUSEHOLD INCOME
KINC CAPITALIST HOUSEHOLD INCOME
UTILW WORKER-HOUSEHOLD INDIRECT UTILITY FUNCTION
UTILK CAPITALIST-HOUSEHOLD INDIRECT UTILITY FUNCTION
DEMANDA HOUSEHOLD DEMAND FOR A GOOD
DEMANDN HOUSEHOLD DEMAND FOR N GOOD
EXPORT EXCESS SUPPLY
IMPORT EXCESS DEMAND
* TRDBAL TRADE BALANCE (omitted!!!, by way of Walras'Law)
```

```
262
263 OBJ OBJECTIVE FUNCTION (TOTAL SOCIAL WELFARE)
264 ;
265
```

Observe the correspondence between the "names" of the EQUATION's and the model description in Table 1 above. Further observe that we have deleted the trade balance equation from the system by using the comment "*" in the first column in line 261. The equation is written only for convenience to the reader and will be ignored by the solver. Furthermore, we know from theory that the value of excess demands vanish in such a system (Walras' law), and the TRDBAL equation is indeed redundant. We will comment more on this issue further below when we introduce the full scale CGE model for Turkey.

The EQUATIONs are specified in the GAMS syntax in lines 269 through 299:

```
265
266
267 ***EQUATION SPECIFICATIONS
268
269 INDEX.. SUM(I, P(I)*0.5) =E= PINDEX;
270
271 PRICE(I).. P(I) =E= PW(I)*ER ;
272
273 PRODUCTION(I).. QS(I) =E= AD(I) * ( ALFA(I)*K(I)**(-RHO(I))
274              + (l-ALFA(I))*LD(I)**(-RHO(I)) )**(-l/RHO(I)) ;
275
276 LABOR(I).. LD(I) =E= ( (P(I)/W)*(l-ALFA(I))*AD(I)**C-RHO(I)) )
277                  **SIGPCI)*QS(I) ; 278
279 LMEQL.. SUM(I, LD(I) ) =E= SUM(HH, LSTAR(HH)) ;
280
Z81 WINC.. Y("WHH") =E= SUMCI, W*LD(I));
282
283 KINC.. Y("KHH") =E= SUMCI, P(I)*QS(I)-W*LD(I) ) ;
284
285 UTILW.. VC"WHH") =E= PROD(I, QD(I,"WHH")**CLES(I,"WHH")) ;
286
287 UTILK.. V("KHH") =E= PROD(I., QD(I,"KHH")**CLES(I,"KHH")) ;
288
289 DEMANDA(HH).. P("A")*QDC"A",HH) =E= CLES("A",HH)*Y(HH) ;
290
291 DEMANDN(HH).. P("N")*QD("N",HH) =E= CLESC"N",HH)*Y(HH) ;
292
293 EXPORT.. QX =E= QS("N") - SUM(HH, QD("N",HH) ) ;
294
295 IMPORT.. QM =E= SUM(HH, QD("A",HH) ) - QS("A") ;
296
297 *TRDBAL.. PW("N")*QX =E= PW("A")*QM ;
298
```

```
299 OBJ.. OMEGA =E= SUM(HH,IG(HH)*V(HH) ) ;
300
```

These correspond to the algebraic specification documented in Table 2.1 above. Observe, however, that we have two extra equations: INDEX (line 269) and OBJ (line 299). The latter one is used by the solver in an attempt to optimize over OMEGA. The INDEX equation defines the normalization rule recognized in the model, something that we have not mentioned for the system so far. Since the excess demand functions, hence the overall real quantities, are homogeneous of degree zero in prices, we have to specify a normalization rule for our system to be able to express the nominal quantities in terms of a "unit of account". In equation INDEX we normalize the system over a price index benchmark, PINDEX, using equal weights for each sector: $\sum P_i w_i =$ PINDEX. In line 315, below, we fix the value of PINDEX at unity to complete the normalization.

```
305
306 *##MODEL RESTRICTIONS
307 *CAPITAL IS SECTOR SPECIFIC
308 K. FX ( I ) = K. L ( I) ;
309
310 *HOUSEHOLDS HAVE NO CONTROL OVER SOCIAL WELFARE WEIGHTS
311 IG.FX(HH) = IG.L(HH) ;
312
313 *##PRICE NORMALIZATION
314 *CHOOSE ONE OF THE FOLLOWING AS NUMERAIRE
315 PINDEX.FX = PINDEX.L;
316 * ER.FX = ER.L ;
317 * P.FX("N")= P.L("N") ;
318
```

Observe that under lines 316 and 317 alternative normalization procedures are proposed, but not implemented. Line 316 treats the exchange rate as the numeraire; whereas 317 fixes the price of N-good, regarding it as the numeraire commodity. Under both cases we would comment on the line 314 using ("*"), thus the overall price level would turn to an "endogenous" variable.

In the GAMS section above, capital stocks are declared *sector-specific* by adding the .FX(I) next to their variable names. Thus, line 308 signifies that the variables K_i are fixed at their level values, K.L(I).

The model is created and declared in line 322 under the name GENEQM. Between slashes, we note that all equations specified thus far are part of the current model:

```
3ı9
320 ***MODEL DEFINITION
321
322 MODEL GENEQM OPEN ECONOMY GENERAL EQUILIBRIUM MODEL IALLI ;
323
```

The model is solved using the SOLVE statement in line 327. GAMS interfaces with the non-linear program (NLP) solver to maximize OMEGA. But as we have noted earlier, the system forming a square block of equations, all given by equality . constraints, stops short of any "optimization" attempt once the constraints are satisfied.

```
326
327 SOLVE GENEQM MAXIMIZING OMEGA USING NLP;
328
329 *SOLUTION VALUES AT TIME 1
330
331 T(TP) = NO;
332 T("ı") = YES ;
333
334 RK( I) = (P.L(I)-QS.L(I)-W.L-LD.L(I)) i K.L(I) ;
335
336 SCALRES("WAGERATE",T) = W.L ;
337 SCALRES("EXPORTS",T) = QX.L;
338 SCALRES("IMPORTS",T) = QM.L;
339 SCALRES("SOCWELFARE",T) = OMEGA.L;
340 SCALRES("RELATlVEPR",T) = P.L("A")/P.L("N") ;
341 SCALRES("EXCRATE",T) = ER.L ;
342
343 PRODUCT("DOMPRICE",I,T) = P.L(I);
344 PRODUCT("WRLDPRICE",I,T)= PW(I) ;
345 PRODUCT("LABDEMAND",I,T) = LD.L(I);
346 PRODUCT("CAPITAL",I,T) = K.L(I);
347 PRODUCT("OUTPUT",I,T) = QS.L(I);
348 PRODUCT("KLRATIO",I,T) = K.L(I)/LD.L(I) ;
349 PRODUCT("PROFITS", I,T) = P.L(I)*QS.L(I)-W.L*LD.L(I) ;
350 PRODUCT( "PROFITRATE" , I, T) = RK( I);
351
```

```
352 HOUSE("INCOME",HH,T) = Y.L(HH);

353 HOUSE("UTILITY",HH,T) = V.L(HH);

354 HOUSE("WLFREWGHTS",HH,T) = IG.L(HH) ;

355 HOUSE("LSTAR",HH,T) = LSTAR(HH);

356 HOUSE("KSTAR",HH,T) = KSTAR(HH);
357
358 QDT(I,HH,T) = QD.L(I,HH);
359
360 DISPLAY SCALRES, PRODUCT, HOUSE, QDT ;
```

We specify the set T as the current period ("1") in lines 332 and 333. The first one says that T does not assume any element of TP, while the next one amends it stating that T takes the value of "1". We calculate the Sectoral profit rates in line 335 as part of preparation for the display tables. We load the solution values to our previously prepared display tables through lines 336 and 359. We document them by using the DISPLAY command in line 360. We next utilize the model in conducting a series of simulation experiments to highlight some further attributes of the Walrasian system at hand.

1-3. Policy Exercises with the Simple Model: Normalization and the Exchange Rate

Given the discussion of the previous section, we first analyze the behavior of the exchange rate in this economy. To do so we perturb the world terms of trade in favor of the A-good by 20%. We trace out the adjustments in the economy under two distinct "normalization" rules: *first,* we choose the N-good as the numeraire of our simple economy (Experiment E1A). Thus, with this specification we set P("N")=PW("N")=1.0. As a second alternative, we continue to utilize the unit-simplex as our overall normalization rule (Experiment E1B).

The overall GAMS program is kept intact as it was introduced above. However, under E1A, the "Price normalization" section is changed to:

```
313
314 *##PRICE NORMALIZATION
315 *CHOOSE ONE OF THE FOLLOWING AS NUMERAIRE
316 * PINDEX.FX = PINDEX.L;
317 * ER.FX = ER.L ;
318 P.FX("N")= P.L("N") ;
319
320
```

Given that P.L("N") = 1.0 from line 114 of the GAMS program, we effectively choose N-good as the numeraire in line 318.

We introduce the experiment as a continuation of the program so that the model "searches" for the new equilibrium starting from the previous solution values:

```
383
384 *INTRODUCE EXPERIMENT HERE: INCREASE PW(A)BY 20%
385 PW("A")=1.2*PW("A") ;
386
387 SOLVE GENEQM MAXIMIZING OMEGA USING NLP;
388
```

The model results are documented in Table 1.3. In what follows we provide a brief depiction of the equilibrium relationships ongoing in our simple economy.

With more favorable terms of trade, we get the Samuelson-Stolper results vis-a-vis to labor incomes. Since the A-good is more labor intensive, labor numerations increase, as employment increases in the production of A. The A-profit rate increases in nominal terms, due to improved terms of trade, while that of N-sector decreases.

With N-price set at unity, we observe no change in the *nominal* exchange rate. The domestic terms of trade are stilI given by the foreign terms of trade, PW("A")/PW("N") = 1.20. It is in fact clear that, under such a specification the exchange rate can play no role in this economy. For, assuming away the transportation costs, the domestic prices are given by:

$$P_i = PW_i ER \qquad (1)$$

thus, the terms of trade become:

$$P_i / P_j = PW_i / PW_j \qquad (2)$$

The exchange rate has no influence in the relative price system of such an economy, and under conditions of balanced trade, it only translates foreign currency units into domestic units.

Now the foregoing results can be contrasted with those of the ElE, where the overall price index was parametric to reflect normalization around the price simplex:

```

*##PRICE NORMALIZATION
*CHOOSE ONE OF THE FOLLOWING AS NUMERAIRE
PINDEX.FX = PINDEX.L;
* ER.FX = ER.L ;
* P.FX("N")= P.L("N") ;


```

So by restricting PINDEX to its level (1.0), we set an aggregate inflation benchmark to the domestic prices. Clearly the domestic prices must now assume different levels than their foreign counterparts in *absolute* values. This is because while the domestic prices satisfy the domestic price level constraint, set at 1.0; the weighted sum of world prices add up to 1.1

($PW_A w_A + PW_N w_N =$ 1.2*0.5+1.0*0.5 = 1.1). The exchange rate must adjust such that:

$\sum P_i w_i$ = PINDEX (3)

where by definition:

$\sum PW_i .ER.w_i$ = PINDEX (4)

Thus, ER satisfies:

ER = PINDEX/ $\sum P_i w_i$ (5)

Observe *from* Table 3 that P_A = 1.091, whereas PW_A = 1.2; and P_N =0.909, with PW_N =1.0. However, the *relative* domestic terms of trade, P_A / P_N is N A N still equal to 1.20 (=1.091/0.909), hence across the two experiments there are no *real effects.* Levels of real production and consumption are the same even though nominal incomes are different. But by the homogeneity of the system, the two solutions are exactly the same given an appropriate choice of the normalization units. The "appropriate" choice of normalization units can be *found* actually by applying the "purchasing-power-parity" price level deflated (PPP-PLD) concept of the exchange rate. Accordingly, by relation (5), the equilibrium value of the exchange rate is 0.909 (=1.0/1.1). That is, as the world "inflation rate" accelerates to 10% in excess over the domestic inflation rate, the nominal exchange rate must *appreciate* by 10% to 0.909, to keep the PPP-PLD exchange rat e constant. The latter is still given by the relative terms of trade, which cannot diverge from the world price ratio by way of small country hypothesis.

Hence in this model, the exchange rate is nothing but a macro index, translating world prices into domestic prices, with no significance on the relative price system nor the real variables.

In the pure trade theory literature, the *real* exchange rate is defined as the "relative value of the basket of traded good prices to the nontraded *(home)* good prices". If we extend our analytical model to include a nontraded, home good, H, the price normalization rule would yield:

$$\sum PW_i.ER.w_i + P_H w_H = \text{PINDEX}. \qquad (6)$$

where the summation is assumed to be over the traded goods, A and N, and

the home good is added with $\sum w_I + w_H = 1.0$.

In this system the relationship between the P_H and the ER becomes:

$$P_H = PINDEX / w_H - (\sum PW_i w_i / w_H) ER$$

Thus,

$$\partial P_H / \partial ER = -(\sum PW_i w_i / w_H) \langle 0$$

Consequently, under the PINDEX normalization rule, nominal changes in the exchange rate will lead to a fall in the price of the home good. What is more, the relationship will be linear given by the term in parenthesis in (7). (Dervis, de Melo & Robinson, 1982, chp 6, further note that, by setting ER to the real exchange rate will itself amount to a normalization rule. This method sets the home good price to the real exchange rate index).

The foregoing discussion stresses the importance of the choice of the normalization rule in the model. It is clear that the normalization procedure is an artifact of the homogeneity of the general equilibrium system, and has no effect on the real quantities. What is important to note is that the nominal variables should always be analyzed in relation to the choice of the numeraire. In particular if a nominal quantity is assumed fixed, that implies fixity *only in terms* of *the numeraire* and one has to be careful in specifying equilibrium relation with the accompanying solution and the numeraire price index.

1-4. The Properties of the General Equilibrium System under Dynamic Adjustment of the Capital Stocks.

Most results of the pure theory of trade (viz. Samuelson, 1949; Stolper and Samuelson, 1941; Rybczynki, 1955) have been derived under conditions of fully flexible factors of production. In this section we illustrate the dynamics of our system with sector specific capital adjusting sluggishly over time in response to the profit rate differentials across sectors. In so doing we will introduce more examples on the GAMS syntax, and reinstate some of the classic propositions of the trade theory which were introduced in the previous section.

We foresee the following adjustment rule for capital over "time":

$$\Delta K(t) = \phi\left(\frac{\pi_i(t) - \pi_j(t)}{\pi_i(t)}\right) K_j(t)$$

$$K_i(t+1) = K_i(t) + \Delta K(t)$$

$$K_j(t+1) = K_j(t) - \Delta K(t)$$

where ϕ is a parameter showing the degree of responsiveness of the capital stock to profit rate, Π_i, differentials.

We implement this specification by using the LOOP and ABORT commands of the GAMS syntax:

```
SET TT(TP) TIME PERIODS FOR DYNAMIC -SOLUTIONS

LOOP (TT,

SOLVE GENEQM MAXIMIZING OMEGA USING NLP;

TCTP) = NO;
TCTT) = YES ;

RK(I) = CP.L(I)-QS.L(I)-W.L-LD.L(I)) i K.L(I) ;
DK = ED*(1- RK("N")/RK("A")) - K.L("N") ;

SCALRES("CAPADlUST",TT) = DK;
SCALRES("WAGERATE" ,TT) = W.L.;
SCALRES("EXPORTS",TT) = QX.L;
SCALRES("IMPORTS",TT) = QM.L;
SCALRES("SOCWELFARE",TT) = OMEGA.L;
SCALRES("RELATIVEPR",TT) = P.L("A")/P.L("N") ;
SCALRES("EXCRATE",TT) = ER.L ;

HOUSE("INCOME",HH,TT) = Y.L(HH);
HOUSE("UTILITY",HH,TT) = V.L(HH);
HOUSE("WlFREWGHTS",HH,TT) = IG.L(HH) ;
HOUSE("LSTAR",HH,TT) = LSTAR(HH);
```

```
322 HOUSE("KSTAR",HH,TT) = KSTAR(HH); "
323 QDT(I,HH,TT) = QD.L(I,HH);
324
325
326 PRODUCT("PRICES",I,TT) = P.L(I); :
327 PRODUCT("LABDEMAND",I,TT) = LD.L(I);
328 PRODUCT("CAPDEMAND",I,TT) = K.L(I);
329 PRODUCT("OUTPUT",I,TT) = QS.L(I);
330 PRODUCT("KLRATIO",I,TT) = K.L(I)/LD.L(I) ;
331 PRODUCT("PROFIIS",I,TT) = P.L(I)-QS.L(I)-W.L-LD.L(I) ;
332 PRODUCT("PROFITRATE",I,TT) = RK(I);
333
334 DISPLAY $(ABS(RK("N")-RK8"A")) LT 0.0005) AD, ALFA, SCALRES, PRODUCT, HOUSE,
335 QDT;
336 ABORT $(ABS(RK("N")-RK("A")) LT 0.0005)
337 "PROFIT RATES ARE EQUALIZED WITHIN TOLERANCE" ;
338
339 *ADJUST CAPITAL STOCKS IN RESPONSE TO PROFIT RATE DIFFERENTIALS
340
341 K.FX("A") = K.L("A") + DK;
342 K.FX("N") = KoL("N") - DK;
343
344
345 *#END OF LOOP
346 );
347
```

Thus we first create a new set TT over the time periods set, TP. Calculate the profit rates; RK(I), in line 307. Capital adjustment is set at line 308, through DK. ED stands for ϕ above, and is set at 1.30:

```
SCALAR DK CAPITAL ADJUSTMENT FROM K("N") TO K( "N" );
       ED ELASTICITY OF CAPITAL ADJUSTMENT /1.30/ ;
```

the LOOP starts at line 300. Un less ABORTED, it continues over again. The ABORT statement at 336 checks whether the absolute levels of the profit rate differentials are less than some 0.0005 units.

If so, the algorithm stops, with the message, "PROFIT RATES ARE EQUALIZED WITHIN TOLERANCE". If not, capital stocks are adjusted in lines 341 and 342, and the LOOP continues once again with a new SOLVE statement in the next "period". The DISPLAY statement precedes the ABORT statement in line 334 because, with ABORT, GAMS stops without any further action. This would leave the equilibrium results unprinted.

Using this apparatus, we illustrate the main propositions of the trade theory in turn.

Table. 1.3 Short Run Adjustments to the 20% Rise in the World Price of A-Good

	Base Value	**N-Good is Numeraire (Exp E1A)**	**Fix PINDEX (Exp E1B)**
Domestic Prices			
Sector A	1.000	1.200	1.091
Sector N	1.000	1.000	0.909
Terms of trade (P_A/P_N)	1.000	1.200	1.200
Aggregate price level			
Domestic	1.000	1.100	1.000
Foreign	1.000	1.100	1.100
Exchange Rate	1.000	1.000	0.909
Labor Employed			
Sector A	3.413	3.818	3.818
Sector N	6.587	6.182	6.182
Real production			
Sector A	2.206	2.319	2.319
Sector N	5.721	5.598	5.598
Wage rate	0.289	0.321	0.291
Profit rate			
Sector A	0.336	0.428	0.390
Sector N	0.336	0.318	0.289
Exports of N	2.480	2.210	2.210
Imports of A	2.480	1.841	1.841