(I)(40 points)

Consider the following private ownership economy: There is only one homogenous good, corn, which is produced using labor and seeds of corn alone. The total corn harvested is given by the (neoclassical) per capita production function:

\[ y = Ak^\alpha \]

where \( k \) denotes capital-labor ratio and \( \alpha = 0.3 \).

The population growth rate is zero in this economy. Bear in mind that in this model, seeds invested in the previous year become capital stock of the current year. Otherwise capital cannot be stored and accumulated. Thus depreciation of capital, \( \delta \) is 1.0. The technology parameter, \( A \), is a constant set equal to 1.

1. Specify the golden rule steady state configuration of this economy under the neoclassical assumptions. That is, find the steady state values of \( k \), \( y \), and consumption per labor, \( c \), for this economy.

2. Suppose now that the economy is characterized by Marxian growth and its savings is given by the classical saving function. Find the wage rate, \( w \); and calculate the net profit rate, \( r \). Is the net profit rate higher or lower than that of the neoclassical configuration? Verify that the unit price equation holds for this economy:

\[ 1 = wl_y + (r + \delta)k_y \]

with \( k_y \) and \( l_y \) denoting the capital-output, and the labor-output ratios, respectively.

3. What is growth rate in the marxian set up? Contrast the Marxian growth rate with that of the neoclassical golden rule steady state equilibrium.

4. Now suppose that the economy is changed to a neo-Ricardian equilibrium. The investment function is given by

\[ g = i(r) = 2r^2 \]

Suppose that the economy continues to operate with the constant \( k_y \) and \( l_y \) ratios as in the neoclassical and Marxian solutions above. Find the new net profit rate, the net growth rate and the wage rate and per capita consumption under the neo-Ricardian equilibrium.
Consider the Chamberlenian endogenous growth model with intertemporal optimization. Suppose that the instantaneous felicity of the representative consumer is given by the following:

\[ U(c) = \int e^{-\rho t} \left[ \frac{c^{1-\theta} - 1}{1-\theta} \right] dt \]

Suppose that population growth rate is zero, and that there is no technological growth. The capital accumulation constraint of the economy is:

\[ \dot{k} = A k^\beta \kappa^\eta - c \]

Assume that \( \beta + \eta = 1 \), and \( \kappa \) denotes aggregate over firms’ capital stocks,

\[ \kappa = \sum k = M k \]

1. Set the consumer’s intertemporal optimization problem as a discounted stream of future consumption profile subject to the capital accumulation constraint. Explicitly state the Hamiltonian of the system and solve for the optimal consumption path. What is the equilibrium rate of growth in this economy? What is the source of positive rate of growth under long run equilibrium in this model?

2. Noting that \( \kappa = M k \), argue that the rate of growth is sensitive to the number of firms.

(III) (20 points)

Consider the following economy:

Private consumption : \( C = 200 + 0.5(Y - T) \)
Net Taxes: \( T = 600 \)
Private Investment: \( I = 1000 \)
Government Expenditures: \( G = 600 \)
Exports: \( X = 500 \)
Imports: \( M = 1000 \)

1. Calculate the equilibrium level of GDP.

2. Suppose that in this economy workers do not save and the saving rate out of profits is 0.75. What is the share of profits and of wages in the aggregate GNP?

3. Now suppose that the investment demand has doubled to 2,000. What is the new equilibrium level of GNP? How is the distribution of income affected?

4. Do you remember this question from somewhere? At least the first part??